Math 2263

Quiz 3

Name Section Score

(5 points) 1. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$2x^2 + 4y^2 + 3z^2 = 1$$

Solution: Take partial derivative of x and y by both sides, we have

$$4x + 6z\frac{\partial z}{\partial x} = 0$$

$$8y + 6z\frac{\partial z}{\partial y} = 0$$

Therefore, $\frac{\partial z}{\partial x} = -\frac{2x}{3z}$, $\frac{\partial z}{\partial x} = -\frac{4y}{3z}$

(5 points) 2. Use differentials to estimate the amount of tin in a closed can with diameter 10 cm and 16 cm if the tin is 0.06 cm thick.

Solution: Since $V = \pi r^2 h$, $dV = 2\pi rhdr + \pi r^2 dh$. By instruction, $r = 5$cm, $h = 16$cm, $dr = 0.06$cm, $dh = 0.12$cm. Therefore

$$dV = 2\pi(5)(16)(0.06) + \pi(5)^2(0.12) = 39.5841$$

(5 points) 3. Explain why the function is differentiable at the given point.

$$f(x, y) = \frac{x}{x+y} \text{ at } (2,\, 3).$$

Solution: $\frac{\partial f}{\partial x} = \frac{y}{(x+y)^2}$, $\frac{\partial f}{\partial y} = -\frac{x}{(x+y)^2}$, it is clear that both f_x and f_y exist near $(2, 3)$, in addition, both f_x and f_y are continuous at $(2, 3)$. Therefore, $f(x, y)$ is differentiable.